One Hundred Years of Elementary School Mathematics in the United States: A Content Analysis and Cognitive Assessment of Textbooks From 1900 to 2000

David Baker
The Pennsylvania State University

Hilary Knipe
New York University

John Collins and Juan Leon
The Pennsylvania State University

Eric Cummings
Cumberland University

Clancy Blair
New York University

David Gamson
The Pennsylvania State University

A content analysis of over 28,000 pages from 141 elementary school mathematics textbooks published between 1900 and 2000 shows that widely used mathematics textbooks have changed substantially. Textbooks from the early part of the century were typically narrow in content but presented substantial amounts of advanced arithmetic and also asked students simultaneously to engage with material in effortful and conceptual ways. A period of change marked the middle of the century, when less advanced topics were presented and problem-solving tasks were simplified. From the mid-1960s onward, however, the trend reversed, and three major changes occurred in primary school mathematics curricula over the next four decades: (a) expansion of topics and the number of pages devoted to each topic; (b) a shift of traditionally more advanced topics from higher to lower grades; and, (c) within arithmetic, an increase in the number, abstraction, and cognitive demand of problem-solving strategies. Implications of these findings are discussed in terms of the historical study of mathematics and curriculum in U.S. schools.

Key words: xxx xxx xxx xxx xxx xxx xxx xxx xxx xxx xxx xxx xxx xxx xxx xxx

This research was supported by a seed research grant from the Social Science Research Institute at the Pennsylvania State University and a grant from the Spencer Foundation. All conclusions are those of the authors and not those of the university or foundation. The authors thank Saamira Halabi, Maryellen Schaub, the editors of JRME, and anonymous reviewers for helpful comments on earlier drafts and John Dossey, Jeremy Kirkpatrick, and Andrew Porter for assistance early in the project.
Elementary School Textbook Analysis

The purpose of this study is to provide thorough empirical evidence of content in elementary school mathematics textbooks in the United States over the course of the 20th century. Arguably, textbooks, or what is referred to in research on mathematics curricula as the *formal* or *written curriculum*, are the most reliable and comprehensive surviving historical record of the U.S. elementary mathematics curriculum (Stein, Remillard, & Smith, 2007). Because of the decentralized nature of curricular adoption and control in the United States and the role of a private market for textbook development, a comprehensive analysis of how mathematics curriculum has changed is not available (Seeley, 2003).

An extensive historical study of textbooks is an important addition to understanding past and present mathematics education. Since the professionalization of U.S. research on mathematics education in the late 1960s, there have been multiple systematic studies of the impact of various curricular innovations and developments on student achievement, all of which have important implications for the practice of mathematics education (e.g., Carroll, 1997; Fuson, Carroll, & Drueck, 2000; Research Advisory Committee of the National Council of Teachers of Mathematics, 1999). Similarly, there is now a literature on the relationship between the written curriculum and a host of characteristics of teachers, students, and classrooms that influence how the written curriculum of textbooks is enacted in the classroom (e.g., Ball & Cohen, 1996; Ben-Peretz, 1990; Cohen, 1990; Lloyd, 1999; for a review, see Remillard, Herbel-Eisenmann, & Lloyd, 2009). Although these important research agendas continue, systematic research on the historical development of written mathematics curriculum as reflected in textbooks has been noticeably absent (for notable exceptions, see Donoghue, 2003; Nicely, 1991; Sinclair, 2008). For the first time in the study of U.S. mathematics education, this research traces the historical development of the nation’s curriculum over the 20th century through a systematic account of primary school mathematics curriculum as reflected in widely used textbooks.

Two key empirical approaches, rooted in prior research on mathematics education, serve as the conceptual perspective for the coding and analysis of U.S. textbooks throughout the 20th century. The first empirical approach is the coding of topical content of elementary school mathematics, such as basic arithmetic, advanced arithmetic, and geometry, including changes in what textbooks present as approaches to problems and related learning exercises for these topics. Content analysis of topical coverage is a widely accepted approach to describing curricula through the content of textbooks for all types of academic subjects, including mathematics (Gehrke, Knapp, & Sirotnik, 1992). This approach is the foundation of notable research projects about the relationship between textbook content and mathematical instruction and achievement, such as the Content Determinants Study (Porter, Floden, Freeman, Schmidt, & Schvville, 1988), the Educational Policy and Practice Study (Wilson, 2003), and the International Survey of Mathematics and Science Opportunities (National Research Council, 1996), as well as sociohistorical studies of school curricula (Meyer, Kamens, & Benavot, 1992).
The second empirical approach assesses historical changes in the cognitive demands of textbooks based on concepts from recent research on human cognitive development and on mathematics education. These concepts include textbook presentations and problems requiring more effortful reasoning, application of novel problem-solving skills, inhibitory control, and use of working memory (Blair, 2006). Psychological research shows that these cognitive processes—often summed up by the term executive functioning—are closely related to the development of more advanced mathematics achievement as well as other skills such as self-regulation; all of which are essential for academic success (Blair & Razza, 2007; Bull & Scerif, 2001; Espy, et al., 2004). There is also neuro-substrata evidence from experiments on the cognitive demand of mathematical reasoning and activation of brain areas associated with executive functioning among school-aged children (Eslinger et al., 2008). Lastly, there is an overlap between the psychological concept of cognitive demand—requiring greater use of executive functioning and less use of automated computation—and research on cognitive demand and mathematics education focusing on curricular and instructional ways that increase elementary school students’ ability to reason about mathematical operations and properties (e.g., Blanton & Kaput, 2000, 2003, 2004, 2005a, 2005b; NCTM, 2000). In the mid-1990s, reform curricula were developed with the goal of increasing the cognitive demand of mathematics tasks as a strategy to improve students’ reasoning with mathematical concepts (Silver & Stein, 1996; Stein, Grover, & Henningsen, 1996). Also, estimates of the cognitive demand of mathematics tasks recently have been used to analyze mathematics curricula (e.g., Cueto, Ramirez, & Leon, 2006; Porter, 2002; Stein & Kim, 2009).

**METHOD**

**Analysis of Textbooks as Indicator of Curricular Change**

Although textbooks are only one part of the mathematics curriculum, research documents that the textbook serves as the cornerstone of the mathematics curriculum in U.S. schools (Clements, 2003; Fey & Graeber, 2003; Seeley, 2003). No direct evidence of what was taught in the nation’s classrooms of the past is available, but textbooks can provide an indication of the intent of the curriculum. School districts now spend, as they have in the past, considerable funds to purchase textbooks, and they would do so only if these books reflect their curricular objectives. For most elementary schools and their teachers, the textbook likely reflects the intended curriculum. Certainly teachers may modify aspects of the content and suggested pedagogy in textbooks, augment with other materials, selectively cover the content, and perhaps never use the purchased text. But for the bulk of the nation’s teachers during each historical period, it seems reasonable to assume that popular texts are a general indicator of the main trends of the elementary school mathematics curriculum. Lastly, because the nation has never had a centralized ministry of education with corresponding curricular records, analyzing archival
artifacts, such as textbooks, is a productive method for studying historical trends in education in the United States (e.g., Cuban, 1984; Tyack & Hansot, 1990).

Textbook Selection

A sample of 141 kindergarten through sixth-grade mathematics textbooks from 33 different series (i.e., sets of texts prepared by the same publisher and designed to span the elementary grades) from 10 different publishing companies from 1904 through 2000 was analyzed. The textbook series, not publishers per se, were selected using the criteria discussed subsequently. The aim in the selection of the sample was to include widely used, popular, commercially produced mathematics textbooks; the complete list of textbooks included in the sample appears in Appendix A.

Initially, publishers’ sales records were examined as a way to estimate the breadth of use of textbooks, but it quickly became clear that historical records of the sales of individual textbook series are mostly unavailable, and when sales records exist, they are not comprehensive, consistent, or complete. Also, publishers’ sales information is proprietary, and aggregated publishers’ records, such as Publishers’ Weekly, do not reliably offer data on the numbers of individual texts sold. Furthermore, reference and historical archive librarians consulted at large universities across the country all agreed that there is no national, historical source on textbooks and their adoption by state and district education authorities. In the absence of sales information and complete national or state adoption records, three sources guided the sampling of textbooks to ensure that the sampled series and their textbooks represented those that had broad distribution, and hence, broad influence on mathematics curricula nationwide.

First, the publication run of a book provided some information on the success of a textbook or series of textbooks. Presumably, a textbook that was poorly received or unused would not have remained in print for an extensive period of time, therefore a multiyear publication run was one criterion used in selecting textbooks and series of textbooks. Elementary and High School Textbooks in Print (Bowker, 1985–1999) lists every textbook in print at the time of each annual volume’s release. Annuals for various years were used to determine the time in print of various textbooks. This publication information was cross-referenced with information from online databases such as WorldCat®. The second source was texts identified by a mathematics education expert as representative of typical books for each decade. Other historical studies that examine a limited numbers of textbooks use knowledgeable experts in the field to guide sampling (Kahn, 1974; Rasch, 1983). The mathematics education expert was presented an initial list of textbook series by

---

1 Institutions included The University of Texas at Austin, Harvard University, Columbia University Teachers College, Stanford University, Indiana State University, and Yale University.

2 John Dossey, Distinguished Professor Emeritus of Mathematics Education at Illinois State University and a former president of the National Council of Teachers of Mathematics, served as the expert.
decade and was asked to verify each series as to its wide distribution and to recommend additional popular series or deletions from the original list. The expert’s opinion matched—with some additions and no deletions—the texts previously identified through multiyear publication runs reported in the Bowker annual survey.

Finally, for earlier periods in the century, available mathematics courses of study from some districts’ and states’ administrative archives were also analyzed for references to required textbooks (see Appendix B for list). For periods later in the century, standards documents from urban and rural districts and states across the country and available state adoption lists were collectively used as an additional cross-reference for the list of modal mathematics texts developed using the first two sources.

Because the goal of the sample is to reflect the most common textbooks used in classrooms, new textbooks at a particular time period aimed at reforming the curriculum, such as the NSF-funded ones of the 1960s and early 1990s, were not analyzed for two reasons. First, these textbooks are often initially limited in their use, and their eventual influence will be reflected later in widely used textbooks (see Stein, Remillard, & Smith, 2007 for a cataloging and discussion of NSF-funded curricular materials). Second, these textbooks contain ideas and approaches that are often substantially different from the commercial textbooks to which most students in the nation were exposed at the time. Although beyond the scope of the present analysis, future analysis could judge the impact of reform texts on subsequent textbooks (for a compiled description of some recent mathematics education reform projects, see Senk & Thompson, 2003).

Textbook Sample as Representative of Textbooks

Because a list of the complete population of textbooks at any given time is unavailable, the sample could not be selected at random. However, considering the extensive methods used to identify widely used textbooks at each time period, a case can be made that the sample is valid as a reasonable representation of the modal textbooks in the nation. But it is not assumed that every school district in the nation always used one of the coded textbooks during each time period. Further, multiple textbooks were selected for each time period. As shown below, although there is much less difference in content from one popular textbook to another than is often assumed, when there were obvious differences among popular textbooks for a period, these were included in the sample to recognize diversity among the most commonly used textbooks. So, the findings represent central tendencies in content across modal textbooks. Textbooks are a major cost for districts, so one can conjecture that poorer districts would be likely to replace textbooks at a slower rate than wealthier districts. Similarly for much of the century, marginalized students—African American students in the south, students who are children of recent immigrants in urban centers—often had only limited access to any textbooks (Walters & James, 1992). Although there are solid reasons to consider the sample of
textbooks and its periodization (see subsequent discussion) a fair picture of content
in most districts, textbook use and distribution among districts serving disadvan-
taged students is a topic ripe for additional research.

Textbook Coding

Given the conceptual perspective previously described, textbooks of a sampled
series were coded in three ways. The first method yielded data for the topical
content analysis approach as well as part of the data on cognitive demand in terms
of reasoning about mathematics. The second and third coding methods yielded data
for assessing historical change in the cognitive demands of textbook material for
use of executive functioning and reasoning skills.

Coding of mathematics topics in first- through sixth-grade textbooks. The frame-
work of topics was developed with knowledge of schemes from past research, but
because most of this research focuses on upper elementary school mathematics
materials, the framework also includes the nature of topical content for lower
elementary grades. To calculate the proportion of a textbook devoted to various
topics and the distribution of topics within the textbook and across the entire series,
every page of every first- through sixth-grade textbook in the sample was assigned
a numeric code from a developed category framework according to its main content.
Here we define topic as a content category in textbooks, and whereas the categories
we have identified may not necessarily conform to boundaries between formal
topics in mathematics, we saw our list as reflecting the content categories suggested
by the organization of the textbooks. Our topic list was developed in two steps.
Applying a “fine grain size” perspective, a master list of topics was compiled in a
prima facie fashion by investigators and coders who then recorded every topic
encountered in a randomly selected subsample of texts from early, mid-, and late-
century.\(^3\) Next, using a larger grain size, coders and investigators condensed the
master list to the final 28 topic-categories framework (listed in the first column of
Figure 1) through trial coding and intercoder reliability checks prior to the start of
the main content analysis.\(^4\)

Page-by-page coding reveals greater detail about the content, how it is presented,
and how students are asked to engage with it than summary codes for chapters or
blocks of pages. Each page of the total corpus of 28,436 pages in all selected text-
books was assigned one of the 28 topic-categories that most accurately described
the presented concepts and tasks on the page. Usually pages did not contain multiple
topics, but when this happened, coders identified the topic with the greatest

\(^3\) Coders were Ph.D. graduate students in education research; one had extensive mathematics ba-
ckground with an undergraduate minor in computation theory, and the other had some undergradu-
ate mathematical training and graduate training up through advanced econometric and psychometric
statistical analyses.

\(^4\) The topics are from our empirical observations of what is presented in the sample textbooks, but,
of course, other organizational schemes are possible.
<table>
<thead>
<tr>
<th>Topics</th>
<th>Reporting topic groups</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number/Counting</td>
<td><strong>Basic arithmetic</strong></td>
</tr>
<tr>
<td>Addition*</td>
<td></td>
</tr>
<tr>
<td>Subtraction*</td>
<td></td>
</tr>
<tr>
<td>Addition/Subtraction*</td>
<td></td>
</tr>
<tr>
<td>Multiplication*</td>
<td></td>
</tr>
<tr>
<td>Division*</td>
<td></td>
</tr>
<tr>
<td>Multiplication/Division*</td>
<td></td>
</tr>
<tr>
<td>General Arithmetic</td>
<td></td>
</tr>
<tr>
<td>Fractions/Mixed Numbers</td>
<td><strong>Advanced arithmetic</strong></td>
</tr>
<tr>
<td>Decimals/Percents</td>
<td></td>
</tr>
<tr>
<td>Ratios/Proportions</td>
<td></td>
</tr>
<tr>
<td>Money</td>
<td></td>
</tr>
<tr>
<td>Time</td>
<td><strong>Geometry/Measurement</strong></td>
</tr>
<tr>
<td>Measurement</td>
<td></td>
</tr>
<tr>
<td>Geometry</td>
<td></td>
</tr>
<tr>
<td>Nonclock Time</td>
<td></td>
</tr>
<tr>
<td>Perimeter/Area/Volume</td>
<td></td>
</tr>
<tr>
<td>Patterns</td>
<td><strong>Reasoning not based in formal mathematics</strong></td>
</tr>
<tr>
<td>Categorization/Grouping</td>
<td></td>
</tr>
<tr>
<td>Informal Geometry</td>
<td><strong>Reasoning based in formal mathematics</strong></td>
</tr>
<tr>
<td>Statistics/Probability/Data Analysis/Graphing</td>
<td></td>
</tr>
<tr>
<td>Informal Algebra</td>
<td></td>
</tr>
<tr>
<td>Number Series</td>
<td></td>
</tr>
<tr>
<td>Grouping Numbers</td>
<td></td>
</tr>
<tr>
<td>Set Theory</td>
<td><strong>Miscellaneous mathematics content</strong></td>
</tr>
<tr>
<td>Digit-Symbol Substitution</td>
<td></td>
</tr>
<tr>
<td>Calculator/Computer</td>
<td></td>
</tr>
<tr>
<td>Miscellaneous</td>
<td></td>
</tr>
</tbody>
</table>

* Indicates operation is performed on whole numbers.

**Figure 1.** Textbook mathematics topics used for coding and reporting topic groups.

coverage on the page. For the relatively infrequent occurrence of a page equally split across topic-categories, coders assigned a half page to each category. Noncontent pages such as tables of contents were excluded, and in the rare case in
which the mathematics material presented on a page did not fit into one of the
categories, the page was coded as “miscellaneous mathematics content.” Coding
data for each textbook and series was entered into a database that was organized
and analyzed by time periods. To summarize effectively the large volume of results
stemming from the topic coding, the 28 topic-categories framework was aggregated
into six reporting topic-groups as shown in the second column of Figure 1:

Basic Arithmetic consists of ideas about number and counting, as well as the
four basic operations of addition, subtraction, multiplication, and division
applied to whole numbers. Arithmetic concepts, ideas, and skills are typi-
cally the earliest taught and generally form the foundation for subsequent
mathematics.

Advanced Arithmetic involves traditional mathematics content utilizing the
four basic operations but is centered on more complex uses and representa-
tions of numbers, including decimals, fractions, and ratios, as well as
percents and proportions. This content often builds on the basic concepts,
ideas, and skills related to operations on whole numbers and is traditionally
taught after basic arithmetic is covered.

Geometry and Measurement includes formal geometric concepts; measure-
ment and units of measure; time; and perimeter, area, and volume. Such
content is often directly applied to daily life in its presentation and use.

Reasoning Based/Not Based in Formal Mathematics consists of informal
algebra, digit-symbol substitution, probability, and set theory. Reasoning
problems from these topic-categories may not have simple algorithmic solu-
tions, or they may require the ability to represent the same number or
problem in multiple forms. Also included in this group are recognition of
sequences of patterns, informal geometry, and grouping/categorization—
areas of mathematics with less mathematical calculation content. All of this
kind of content was judged to require effortful reasoning, application of
novel problem-solving skills, inhibitory control, and use of working memory.
For example, recognition of sequences of patterns often involves no numbers
or arithmetic operations but demands skills such as attention shifting and
working memory. For clearer displays of results, this reporting group
combines all reasoning categories shown in Figure 1.

Miscellaneous Nonspecific Mathematics Content is mathematics material
that falls outside the 28 topic-categories.

---

5 Review and evaluation pages were coded as separate categories to show how much the emphasis on
testing shifted, but are not included as one of the 28 topic-categories and are not reported on here.
To understand the shifts over time at which grade topics were introduced, selected topic-categories were traced throughout all series to track at which grade and at which time period material first appeared. For example, every appearance of a basic arithmetic topic-category across an entire textbook series was recorded and noted in terms of its proportional distance into the textbook to account for changing size of textbooks. These data produce a detailed picture of shifts both within and across content areas in terms of what students were expected to master in different grades during different historical periods.

Coding of mathematics topics in first- through sixth-grade textbooks. As a way to learn mathematics, textbooks often present problem-solving strategies. Thus, problem-solving strategy coding allows for examination of shifts in the ways students are asked to interact with mathematical content. During the process of page-by-page topic-category coding of first-grade and fourth-grade textbooks, all presented problem-solving strategies for basic arithmetic topic-categories were recorded and coded. Problem-solving coding was limited to basic arithmetic, because basic arithmetic is found in all textbooks in all time periods, is fundamental to advanced topics, and is taught regardless of how much content was added to the curriculum in later time periods. First-grade and fourth-grade textbooks were chosen as representative of the early and middle elementary grades. For most of the century, first grade marks the beginning of basic arithmetic, and by fourth grade all basic operations have been introduced, as have problem-solving strategies for using basic operations and basic arithmetic.

A problem-solving strategy was defined as any presented method to solve a problem, and the coding was completed in two parts. First, the frequency of each kind of problem-solving strategy was recorded for each textbook. Second, each kind of problem-solving strategy was coded to indicate the level of abstraction about mathematics that seemed to be required of the student to use the strategy effectively. Developed through an iterative process, the mathematics abstraction scale ranges from problem-solving strategies that seemed chiefly concrete/rote to those that seemed to require abstract/conceptual application of mathematics. First, a preliminary subsample of problem-solving strategies found in textbooks across the century was selected, and to avoid any bias from preconceived notions of the nature of textbook material in certain historical periods by coders, the strategies were stripped of any information that identified the time period of their use in textbooks. Next, trained project coders developed working definitions of abstraction.

---

6 For example, if a book had only 100 pages, page 50 would be at the 50% point in that particular book, but if another book had 200 pages, page 50 would be only at the 25% point in that textbook. In this way, it was simple to compare how far into a text for a particular grade a specific skill or topic was introduced.

7 For the purpose of this research, algorithms were not considered strategies. Algorithms are characterized by procedural application of set rules, whereas strategies are characterized by more conceptual application of arithmetical properties of number and allow for multiple means to solve a problem. Like strategies, algorithms can vary in their level of conceptualization or abstraction depending on how much instructional text surrounds the algorithmic demonstration of problem solving.
tion levels that were then used and tested for interrater reliability in trial coding of the preliminary set of strategies. This process continued until a reliable scale emerged, as described in Figure 2 (see subsequent discussion for estimates of interrater reliability).  

<table>
<thead>
<tr>
<th>More concrete/rote</th>
<th>General strategy description</th>
<th>Strategy examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Nonconceptual strategy</td>
<td>Memorizing addition table</td>
</tr>
<tr>
<td>1 Physical</td>
<td>Representation and counting</td>
<td>Counting all with pictures or objects, either statically or dynamically</td>
</tr>
<tr>
<td>2 Representation</td>
<td>with fairly concrete charts, symbols, etc.</td>
<td>Using manipulatives: counters, blocks, cubes, pictures</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Counting on and counting back</td>
</tr>
<tr>
<td>3 Moderately</td>
<td>Abstract representation</td>
<td>Using place-value chart to add/subtract multidigit numbers</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Renaming using picture representation of grouping to 10s, 100s, etc.</td>
</tr>
<tr>
<td>4 Conceptually</td>
<td>Based shortcuts for easier mental calculation</td>
<td>Compensating (e.g., 13 – 9 ≥ 10 – 9; 1 + 3 = 4, so 13 – 9 = 4)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Using easier or familiar facts to solve more difficult problems</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Estimating to check answers and/or approximate answers</td>
</tr>
<tr>
<td>5 Conceptual use</td>
<td>of properties of operation and number to facilitate calculation</td>
<td>Using commutative property of addition or multiplication. If a fact is known, its reverse is known.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Use of reverse operation to check answer</td>
</tr>
</tbody>
</table>

Figure 2. Coding scale of mathematical abstraction of basic arithmetic problem-solving strategies in first- and fourth-grade textbooks.

Recognizing that abstraction levels of presented strategies could be altered some by teachers using the textbook, the problem-solving strategy coding was done under the assumption that what was presented in the textbook was the intended approach. Combined with the frequency of types of strategies, the coding here represents the modal opportunities for teachers to teach the material in the way intended by the textbook.
Textbook material with high demand for problem solving can include either high or low amounts of mathematical computation. For example, higher order cognitive skills and fewer mathematical computation skills are required of new learners of informal geometry, particularly mental rotation skills and recognition of sequences of shapes or objects, whereas higher order cognitive skills are required for the more overtly mathematical tasks of data representation and reasoning about probabilities and estimation. Also included here are problem-solving strategies that require the application of more abstract mathematical principles as well as textbook exercises requiring multiple problem-solving strategies and multiple representations of the same problem. The opposite of this kind of material would be the relatively low amount of working memory, reasoning, and new problem-solving cognitive skills needed for textbook exercises entailing rote memorization of mathematical facts and repetitive calculation using a presented algorithm.

**Coding learning tasks for all mathematics topics.** In addition to presenting the student with problem-solving strategies with varying levels of mathematical abstraction, textbooks also often present tasks (i.e., problems for students to solve and computational tasks for the student to complete) to assist the student in mathematics learning. A preliminary analysis of textbooks showed a range in these tasks from memorizing mathematics facts and practicing calculation using an algorithm to estimation and mental mathematics. Because of the massive volume of tasks in textbooks, a 20% random representative sample of pages was selected for each topic of the 28 topic-categories framework (total subsample of over 1,600 pages). An exhaustive list of types of task—algorithms, strategies, repetitive drill, estimation, self-checks, and mental math—was established from an examination of all textbooks. Next, using preliminary examples of tasks, trained project coders were tested for interrater reliability. Then using this list of task types, each page in this subsample was coded as to the frequency of each type of mathematics task. For presentation of these results, types of tasks are aggregated into two categories.

The first category includes all instances of tasks requiring a high degree of effortful cognitive activity: those requiring students to estimate, perform self-checks of answers, or carry out mental mathematics and reasoning. Self-checking tasks ask students to examine the reasoning behind their work and check its logic and accuracy in solving a problem. There are some activities that could overlap between estimation and mental mathematics, because some forms of estimation can be performed using mental mathematics, but since calculations for estimation could be done by hand, on a calculator, or by mental calculation, estimation is not always mental mathematics. The idea in estimation is to find ways to reach an approximate answer for the purpose of checking whether an answer is reasonable or finding a quick answer when the exact calculation is not necessary. It is also used

---

9 Sampling at the rate of 20% was chosen through statistical comparisons of the coding of randomly selected pages to the coding of all pages, and 20% was the smallest sample that retained acceptable representation.
to help demonstrate concepts about number, such as place value. On the other hand, mental mathematics refers specifically to any mathematical activity that requires students to calculate using mathematical operations completely “in their heads.”

The second category includes all instances of tasks requiring little effortful reasoning, such as tasks requiring rote calculation using a given algorithm. Different types of algorithms presented on a page were also recorded.

**Periodization**

Sampled textbook series were initially arranged by decade, but decades reflected neither well-established periods of curricular reform nor the market availability of textbook series as shown in publication runs. Commonly used textbook series publication runs tended to cluster together over years, and then most tended to end publication at about the same time. Therefore, we use time periods that reflect cycles of publication runs and that also have the most consistency across textbook series within intervals. The time period is the main unit of analysis by which results are presented below, and textbook series are aggregated to indicate curricular trends during each time period.

The substantive time periods are the following (with the number of widely used textbooks coded, first through third grade, fourth through sixth grade): 1904–1921 (11, 12), 1924–1931 (9, 6), 1932–1948 (10, 12), 1948–1963 (19, 15), 1964–1971 (6, 7), 1974–1978 (6, 6), 1981–1991 (6, 6), 1991–1999 (6, 6). Nearly every time period is separated by a clean break in publication dates, suggesting that the periods may align with significant periods of curricular and educational change; therefore, to remain competitive in the market, most major publishers tended to release a new textbook series to match the new expectations. To be conservative in estimating patterns across time periods, there are gaps between some periods because no published texts met the sampling criteria. The calculated coefficients of variation (CV) across textbooks in topical coverage for each period demonstrate that variation within periods is minimal, indicating reasonable content uniformity within periods (see bottom of Figures 3 and 4 for reported CVs per time period).

---

10 Examples of each are: Estimation without mental mathematics: Round to the nearest hundred and add to find an estimate, then calculate the exact answer and compare: 4378 + 5241. Mental mathematics with estimation: In your head, estimate the difference between 89 and 41 (usually the strategy is to round to the nearest 10 and subtract). Mental mathematics, but not estimation: Find the following sums without pencil and paper: 4 + 8, 7 + 2, 3 + 6, . . .

11 In cases for which the last year of one time period is the same as the first year of the next period, there were books whose print runs ended in the final year of a period and others whose print runs began in the same year. Books with final prints in that year were placed in the earlier period and those with first prints in that year were placed in the later period.

12 Although some overlap is still likely because textbooks printed in any given year were likely used for several years afterward, because all periodization is based on publication and print information, the effects of such lags should be spread equally among all time periods in the century.
Selection of time periods was determined by modal breaks in publication runs across sampled textbook series that clustered together at various points over the century, with a time period ending when most sampled series in a cluster went out of publication. For example, starting in 1904, 11 commonly used first- through third-grade series were in publication until 1921, by which time most publication runs ended. This method determined the time period placement of the majority of sampled textbook series, but not all textbook series in a cluster had publication runs that ended at precisely the same time. Some series spanned several time periods, contributing to the nation’s curriculum over a longer part of the century, and sometimes the publisher revised a series with a new edition extending its publication run but with significantly changed content. To handle these realities of textbook series publishing, after the modal breaks in publication runs were determined, each textbook series was individually evaluated concerning the time period(s) in which to place it as follows. Information was acquired on the complete publication run for every series in the sample. Annual versions of each edition of a series were compared page-by-page to determine whether the textbook series remained the same for its entire publication run. As was most frequent, when the content was found to be the same, the series was included in the time period that best matched its publication run. Whenever possible, textbook series were placed into one time period, but if a series with the same content remained popular across a significant portion of the subsequent time period its data were included in both time periods. In the rare case when the content of a series had significantly changed within a time period and the series remained popular throughout, it was treated as two unique series to capture the changes in the time period. If a series was significantly revised at the end of a time period and remained popular, the revised version was placed into the next time period.

Coder Training and Reliability of Textbook Coding

A detailed coder manual was developed and used to train all the main coders. The manual gave specific definitions for the 28 topic-categories and rules and definitions for coding problem-solving strategies and mathematical tasks. Interrater reliability was tested on an independent coding of 20% of the sample across all three types of material (i.e., topics, problem-solving strategies, and tasks) by each trained coder. Interrater reliability was generally high for all three types of coded material (Spearman’s Rho values between 0.7 and 0.98). Further, Wilcoxon signed ranks tests indicated no significant differences between coders on any aspects of the coding. In the few instances of lower reliability (below 0.7), the coders returned to the 20% sample and reviewed discrepancies until interrater agreement was reached, and coding rules and definitions were subsequently modified for all textbooks in the sample.

Grade Aggregation

Analysis of multiple textbooks across eight historical periods and six grades yielded a tremendous amount of data. To summarize these findings parsimoniously,
various grade aggregations were essential. But, in each case, a full analysis of textbooks for each grade was conducted to verify that the overall aggregated trends reported are consistent with individual grade-by-period analyses. Any divergent individual grade findings are specifically noted.\textsuperscript{13}

\textbf{RESULTS}

The findings across the entire century break into three metatrends. The first two stem from the content analysis and the last from the cognitive assessment: Metatrend I, expansion of curricular topics; Metatrend II, historical patterns of expansion of curricular topics across grades; and, Metatrend III, shifts in cognitive demand of problem-solving strategies and learning tasks. Each metatrend has several components.

\textit{Metatrend I: Expansion of Curricular Topics}

In size alone, elementary school mathematics textbooks underwent substantial growth, measured both by physical dimensions and total number of pages. Textbooks in the earliest period (1904–1921) contained a mean of 87 content pages, whereas textbooks in the 1990s had a mean of 330 content pages, a growth of almost 300\% (mean page change, $t = 19.8$, $p < .05$). Along with this prodigious growth in length, mathematics textbooks grew with regard to the number of content categories and the number of pages devoted to these content categories.

Changes across the 20th century in mathematics topics are represented in two ways: (a) the average number of pages for each of the five reporting topic-groups in Figure 3 (Panel A for first through third grade and Panel B for fourth through sixth grade), and (b) the average percentage of the textbook dedicated to each topic group in Figure 4 (Panel A for first through third grade and Panel B for fourth through sixth grade). The proportion of the textbook used to cover a topic is a traditional indicator of curricular distribution, but because the size of textbooks has grown so dramatically over the century, the number of pages per topic is also presented in parentheses as an indicator of changes with $t$-test values ($p \leq .05$) of changes in pages over noted time periods. There are three interrelated components to the trend in the distribution of the content within the expanding mathematics textbook.

\textit{I-A. The early dominance of basic arithmetic.} The distribution of content focusing on basic arithmetic in textbooks changed over the century. As shown in the first part of the columns in Figures 3 and 4, first- through third-grade textbooks at the beginning of the century were dominated by basic arithmetic content, whereas fourth- through sixth-grade textbooks were made up of basic mathematics, advanced mathematics, and geometry/measurement, each having a substantial

\textsuperscript{13} Disaggregated results by grade are available from the authors upon request.
Panel A. First Grade Through Third Grade

Panel B. Fourth Grade Through Sixth Grade

Figure 3. Number of content pages in textbooks, 1904–2000.
Figure 4. Percent of content pages in textbooks, 1904–2000.

Note. See Figure 3 for the number of books and the coefficient of variation for each time period.
share. Then, from 1924 to 1963, a substantial change occurred, and curriculum became considerably more focused on basic arithmetic. For example, at the beginning of the century the average first- through third-grade textbook devoted 72% (or 50 of 70 pages) to basic arithmetic, but by the late 1920s this increased to 85% of a larger text that nearly doubled the number of pages on basic arithmetic (50 to 110 pages; \( t = 2.12, p < .05 \)). For fourth- through sixth-grade texts, the proportion of basic arithmetic increased modestly from 46% to 57%, but because of the expanding size of textbooks, the number of basic arithmetic pages more than doubled (48 to 101 pages; \( t = 1.8, p = .087 \)) by the late 1920s. This trend intensified over the next decades so that by the middle of the century the majority of first- through third-grade textbooks were devoted to topics of basic arithmetic (85%, 110 to 143 pages; \( t = 4.6, p < .05 \)), and the majority of content of fourth- through sixth-grade textbooks continued to focus on basic arithmetic (65%, 101 to 159 pages; \( t = 7.4, p < .05 \)).

From the 1960s onward, the dominance of basic arithmetic in fourth- through sixth-grade textbooks leveled off and even diminished. Basic arithmetic dropped to 41% (159 to 113 pages, \( t = 5.1, p < .05 \)) during the 1964–1971 period and maintained this approximate proportion along with a small increase in pages until the end of the century. The historical pattern is similar but less pronounced for first- through third-grade textbooks, for which the proportion of basic arithmetic declined in the 1964–1971 period, starting at 75% and then dropping over the next three periods to about two-thirds by the 1990s.

**I-B. Expanding advanced arithmetic and geometry/measurement 1964–1999.**

Early in the century, first- through third-grade textbooks contained 10% advanced arithmetic or just 7 pages, and this proportion shrank over the next three time periods to virtually no coverage. Coverage of geometry/measurement also declined in proportion to other content areas and only increased slightly in number of pages up to midcentury. Although from the beginning of the century these two content areas are a larger proportion of fourth- through sixth-grade textbooks, there is a similar decline in proportion and only a modest increase in total pages up to the mid-1960s. This trend changes for both grade groups of textbooks from the mid-1960s through the end of the century. In first- through third-grade textbooks, advanced arithmetic expanded from 2% to 3% (3 to 7 pages, \( t = 1.7, p = .110 \)) before remaining at about 5% (16 pages by 1990s) for the rest of the century. In these lower grades, the topics of geometry/measurement expanded from 9% at midcentury to 17% (15 to 54 pages, \( t = 8.3, p < .05 \)) by century’s end, representing a major growth in number of pages. In fourth- through sixth-grade textbooks, the proportion of advanced mathematics expanded modestly, but the growing textbook yielded a 40% increase in content pages from midcentury to the last period of the century (44 to 71 pages, \( t = -5.38, p < .05 \)). Also in the advanced grades, the topics of geometry/measurement expanded to account for roughly one fifth of all content by the mid-1960s, a doubling of pages (29 to 60, \( t = 4.23, p > .05 \)).
I-C. Rise of reasoning-based content. Prominent among this expansion of topics are those requiring reasoning, either with or without formal mathematics. Until the mid-1960s this kind of content was rare, covering barely 3% (2 pages) of first-through third-grade textbooks and at most 5% (5 or 6 pages) of fourth-through sixth-grade textbooks. Yet by the end of the century, first- through third-grade students used textbooks that had on average 12% (2 to 39 pages, $t = 2.3, p < .05$) reasoning content and fourth- through sixth-grade students learned from textbooks that had 17% (5 to 60 pages, $t = 6.1, p < .05$) reasoning content.

Panel A of Figure 5 shows an example of a typical late-century reasoning lesson with considerable formal mathematical content in a fourth-grade textbook from the series *Mathematics: The Path to Math Success!* (Cavanagh et al., 1999). The learner is asked to make a bar graph of two tables of data, and report an analysis across three dimensions of the data—an exercise that requires reasoning and application of statistical concepts to analyze the data. Panel B of Figure 5 is a reasoning task from the same textbook without formal geometry content. Here, the learner is asked to exercise mental rotation skill in the context of transformation geometry. Similar to the statistical task, it is cognitively demanding in the sense that it requires significant, effortful processing to hold in mind the given information and shift attention between the provided information and the goal. These types of tasks represent an increasing proportion of the curriculum from the mid-1960s onward.

Last, in addition to increases in reasoning-based content, a significant feature of textbooks from the mid-1960s onward is the inclusion of topics outside arithmetic and geometry/measurement. For example, the average number of individual topic-categories covered in textbooks increased by about 50% from the first decade to the last decade: over the course of the century, first- through third-grade textbooks went from covering roughly 11 topics to covering over 16 ($t = 3.56, p > .05$), and fourth- through sixth-grade textbooks went from covering 14 topics to covering over 20 ($t = 3.23, p > .05$).

Metatrend II: Historical Patterns of Expansion of Curricular Topics Across Grades

There are two broad historical patterns behind the expansion of primary school mathematics as reflected in textbooks. The first pattern is the introduction and steady growth in the use of mathematics textbooks in early elementary grades over the 20th century. For example, in the first decade of the 1900s, mathematics was rarely taught before the second grade, and some educators thought it “unwise to place an arithmetic [textbook] in the hands” of a child too soon (Wentworth, 1907; see also Michalowicz & Howard, 2003, who analyzed 19th-century mathematics textbooks). In the 1930s and 1940s, the small proportion of children who attended kindergarten received no mathematics textbooks, and mathematics instruction often began during the second half of first grade. Yet, by the 1950s and 1960s, first-grade textbooks were standard and contained increasingly more mathematical content.
Figure 5. Two examples of reasoning exercises from 1999 textbooks with high and low formal mathematics content.
By 1975, most children attended kindergarten and used textbooks that covered 10 separate mathematics topic-categories with an average of almost 9 pages devoted to each category. And in the 1990s, kindergarten textbooks covered more content categories (14) on more pages per category (12.93) than the texts given to second graders in the 1930s ($t = 1.47, p = .146$).

The second broad historical pattern of this metatrend is findings from the 1960s onward of more advanced mathematics topics and new topics introduced in textbooks for ever-earlier grades. The textbook analysis revealed three different ways this occurred, all resulting in advanced and new topics shifting to earlier grades over time: (a) gradual shifts in topic placement, typically resulting in earlier introduction of content by century’s end; (b) rapid change in the introduction grade of a topic, such that it was introduced several grades earlier; and (c) sudden introduction to most or all grades of a new topic that has not been found in textbooks of any preceding time period.

II-A. Gradual shifts in topic placement. By the 1960s, topics that had been introduced in higher grades began to be introduced earlier than in preceding time periods. This was the opposite of what had occurred from 1920 until the 1950s, when introduction of some content drifted up to higher grades. For example, as demonstrated in Panel A of Table 1, content focusing on decimals and percents was introduced in fourth-grade textbooks at the beginning of the century. From 1924 to 1963, the topic was not introduced until fifth grade, but by 1981 this material was introduced in third-grade textbooks.

The grade of introduction for other advanced arithmetic content follows a similar pattern. For example, during the first half of the century there was a decrease in coverage of fractions and it was not introduced in lower grades. Beginning in the 1960s, however, fractions were taught earlier and in greater proportion in most books, resulting in an even greater increase in coverage of this content. The idea of a fraction came to be introduced in kindergarten by the 1990s, whereas in the earliest time periods they were typically not covered until third or fourth grade. This same process resulted in basic arithmetic shifting down to the lowest grades; for example, subtracting single-digit numbers moved from being introduced in the second grade during the period from 1911 to 1930 to becoming standard kindergarten textbook content by 1999.

II-B. Rapid pushdown of content. In later time periods, content moved more quickly from higher to lower elementary grades and even to kindergarten. As shown in Panel B of Table 1, beginning in the mid-1970s, material on statistics, probability, and data analysis was suddenly introduced in kindergarten and first-grade textbooks instead of fourth-grade, as was the prior practice. Simultaneously, as a topic is

---

14 Kindergarten mathematics textbooks did not start to appear until the 1970s, so they were not included in the initial content analysis portion of the study, but from 1970 onward, similar coding of widely used kindergarten textbooks helps to inform the changes in recent decades for the youngest of students.
Table 1  
*Shifts of Mathematics Topics to Earlier Grades*

Panel A. Proportion of text devoted to lessons on decimals and percentages (advanced arithmetic) by grade and time period

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>30.44</td>
<td>28.95</td>
<td>19.38</td>
<td>9.64</td>
<td>11.61</td>
<td>17.33</td>
<td>16.61</td>
<td>15.63</td>
</tr>
<tr>
<td>5</td>
<td>8.04</td>
<td>10.73</td>
<td>7.94</td>
<td>3.88</td>
<td>4.46</td>
<td>7.86</td>
<td>9.88</td>
<td>8.97</td>
</tr>
<tr>
<td>4</td>
<td>3.96</td>
<td>0a</td>
<td>0.19</td>
<td>0</td>
<td>1.68</td>
<td>2.47</td>
<td>3.69</td>
<td>6.17</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.39</td>
<td>2.40</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>Xb</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Panel B: Proportion of text devoted to lessons on statistics, probability, or data by grade and time period

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0.39</td>
<td>0.72</td>
<td>3.99</td>
<td>4.39</td>
<td>6.86</td>
<td>4.98</td>
<td>6.51</td>
<td>12.23</td>
</tr>
<tr>
<td>5</td>
<td>1.19</td>
<td>0.62</td>
<td>3.79</td>
<td>2.65</td>
<td>2.22</td>
<td>5.86</td>
<td>6.52</td>
<td>13.06</td>
</tr>
<tr>
<td>4</td>
<td>0.21</td>
<td>4.90</td>
<td>2.82</td>
<td>1.92</td>
<td>1.87</td>
<td>4.64</td>
<td>5.46</td>
<td>9.74</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.07</td>
<td>0.49</td>
<td>2.52</td>
<td>4.01</td>
<td>8.55</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2.98</td>
<td>1.95</td>
<td>3.28</td>
</tr>
<tr>
<td>1</td>
<td>X</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2.94</td>
<td>1.34</td>
<td>3.57</td>
</tr>
<tr>
<td>0</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>1.71</td>
<td>0.32</td>
<td>5.60</td>
<td></td>
</tr>
</tbody>
</table>

Panel C: Proportion of text devoted to lessons on informal geometry by grade and time period

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
<td>0.22</td>
<td>0.60</td>
<td>0.58</td>
<td>1.12</td>
<td>1.63</td>
<td>1.92</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.21</td>
<td>1.20</td>
<td>1.46</td>
<td>2.68</td>
<td>2.95</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0.36</td>
<td>0.52</td>
<td>0.40</td>
<td>2.57</td>
<td>3.39</td>
<td>2.48</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0.14</td>
<td>0.08</td>
<td>0</td>
<td>1.63</td>
<td>2.99</td>
<td>3.73</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0.46</td>
<td>0</td>
<td>0.09</td>
<td>0</td>
<td>3.17</td>
<td>1.99</td>
<td>3.60</td>
</tr>
<tr>
<td>1</td>
<td>X</td>
<td>0</td>
<td>0</td>
<td>0.42</td>
<td>0</td>
<td>2.37</td>
<td>2.91</td>
<td>5.05</td>
</tr>
<tr>
<td>0</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>5.50</td>
<td>8.88</td>
<td>7.05</td>
</tr>
</tbody>
</table>

*a* 0 = no coverage at that grade  
*b* X = no available textbooks
introduced in earlier grades in a single time period and in subsequent periods, a
greater portion of these lower grade textbooks are devoted to that topic.

**II-C. Sudden inclusion of new content.** Finally, new mathematics topics that were
not present in any grade in the preceding period are suddenly introduced in text-
books for all or most grades. A good example of this phenomenon appears in Panel
B of Table 1 for informal geometry. It was virtually not taught in any grade in any
time period until the mid-1970s, when some informal geometry content appeared
in textbooks for grades K–6. Also, by 1991, properties of geometric figures form
a substantial portion of the content in a widely used kindergarten textbook, taking
up about 9% of the text (Eicholz, 1991), whereas in the two earlier time periods,
only 2 or 3 pages were typically devoted to any sort of geometry in kindergarten
(3 to 30 pages, \( t = 4.78, p > .05 \)). The nature of the content changed dramatically
as well. Early grade textbooks in the 1920s infrequently presented informal geom-
etry, and if they did, it was a simple introduction to the concept of shapes and plane
figures (see example in Figure 6, Panel A). By the 1990s, first- and second-grade
textbooks asked students to solve geometry problems that require mental rotation
and the translation of three-dimensional shapes to plane figures (as shown in Panel
B of Figure 6).

**Metatrend III: Growing Cognitive Demand of Problem-Solving Strategies and
Learning Exercises**

Results of the problem-solving strategy coding indicate that although basic
arithmetic forms the foundation of elementary school mathematics, the material
on basic operations changed substantially over the course of the century.
Increasingly, students are asked to engage in more frequent effortful reasoning-
based problem solving with more complex strategies as opposed to using a memo-
rized computational approach. Like the previous metatrends, from the 1920s to the
early 1960s, textbooks presented fewer unique problem-solving strategies and more
mechanical drill. Then, starting in the mid-1960s, there was significant growth in
effortful reasoning-based problem solving and exercises until the end of the century.
This change from the mid-1960s is made up of four components: (a) the expansion
of types of problem-solving strategies presented, (b) increasing abstraction and
conceptual focus of problems and solution strategies, (c) growth of arithmetic
exercises using reasoning, and (d) a continuation of calculation and drill in basic
arithmetic.

**III-A. Expansion of problem-solving strategies.** As shown in the dashed trend line
in Figure 7, textbooks in the 1990s presented an average of 20 different strategies
(see Figure 8 for examples of different types) for solving basic arithmetic problems
that required addition, subtraction, multiplication, or division, which is nearly
quadruple the number students were exposed to in textbooks published between
1904 and 1921 (5.2 to 20, \( t = 3.06, p < .05 \)). However, this change did not occur in
a completely linear fashion. In the earliest period, students were exposed to very few strategies, but in the 1924–1931 period, the number of strategies provided increased to an average of 10.5. By the 1932–1948 and 1948–1963 periods, though, fewer strategies for basic arithmetic were provided than in the preceding time period. Then, beginning with the 1964–1971 period, there was a precipitous increase in number of strategies followed by a dip in the 1980s, and, by the 1990s, a return to the levels of the 1960s.

As an illustration of this increase in number of strategies, consider differences in how multiplication is presented in two fourth-grade texts published 30 years apart, *Making Sure of Arithmetic* (Morton, Gray, Springstun, Schaaf, & Rosskopf, 1958), and *Houghton Mifflin Mathematics* (Capps, 1989). In the 1958 textbook, students are presented with three sets of instructions corresponding to different
Panel B: 1999.

From Silver Burdett Ginn Mathematics Student Edition Grade 2. Copyright © 1999 by Silver Burdett Ginn. Used by permission of Pearson Education, Inc. All rights reserved.

Figure 6. Comparison of informal geometry content in first and second grades in 1929 and 1991.
strategies for solving multiplication problems: repeated addition \((32 + 32 + 32 = 3 \times 32)\), using related number facts to facilitate computation, and using arrays. In the 1987 textbook, students are presented with five sets of instructions related to different strategies for solving similar multiplication problems: using common multiples of numbers to facilitate computation; using zero place holders when multiplying by 10, 100, and so forth; estimating products; using place-value figures for multi-digit multiplication; and using a method for guessing, checking, and adjusting quotients to solve a problem. By 1999, a single textbook page might remind students that there are 6 or more strategies for solving arithmetic problems. In one fourth-grade text (Ginsburg, 1999), there were 26 such pages reminding students of between 2 and 8 possible strategies for solving the type of problem presented. The culmination of this trend by the end of the century results in an even greater number of distinct problem-solving strategies than presented during any other period, as well as a heavier emphasis on use of multiple strategies.

### III-B. Increasing abstraction and conceptual focus.

Along with the expansion in types of problem-solving strategies presented in textbooks, rote and mechanical exercises are replaced by more abstract conceptual problem-solving strategies (see Figure 2 for abstraction levels). The solid trend line of the mean level of abstraction of presented strategies in Figure 7 shows a steady, consistent trend toward strategies

---

**Figure 7.** Average number of different kinds of problem-solving strategies (dashed line) and average level of abstraction of problem-solving strategies (solid line) for basic arithmetic in first and fourth grade combined.
that require more conceptual understanding and flexible representation of number and operation. With the exception of the outlier spike during the 1964–1971 period, the trend steadily increases (abstraction level of 1.5 to 2.9, $t = 4.35$, $p > .05$).

In contrast to the later abstraction of problem-solving strategies involving conceptually based shortcuts for doing mental mathematics and the conceptual use of mathematical properties such as the commutative property, earlier textbooks often provided pages of many facts, such as $12 + 5 = 17$, $12 + 6 = 18$, and $12 + 7 = 19$, and then asked students to review the problems repeatedly until they could recite them accurately from memory under a specific time constraint. Although these same early textbooks demonstrated mathematical properties to students, they did so only after students had spent considerable time memorizing. By the middle of the century, the same types of mathematics tasks were frequently represented with pictures of groups
of objects for students to count. Students were shown how to represent tens and ones with counters (usually sticks and bundles of sticks), or they were shown the algorithm for adding vertically and carrying into the tens column. However, unlike early and later textbooks, these midcentury textbooks rarely, if ever, provided a way for the student to understand the mathematical properties behind the illustrations, such as the idea that the sticks and bundles represent the place values of digits in base-10 representations of numbers. Instead, these books often provided explicit, step-by-step instructions for carrying out mechanical computation. Because these algorithms were largely mechanical and had little, if any, conceptual explanation, they made minimal reasoning demands on the problem solver.

Comparatively, textbooks from the mid-1960s onward presented more problem-solving strategies requiring the use and mastery of field properties for real numbers (often in the context of whole numbers for the youngest students) and/or the application of multiple previously learned strategies to achieve a new and deeper understanding. Although these later textbooks continued to include the types of strategies found in textbooks from the first three historical periods, additional new problem-solving strategies required students to use field properties (e.g., commutative property of addition for real numbers) to maximize their existing knowledge or decompose more difficult problems into familiar ones. By the 1990s, the abstraction level of the average presented problem-solving strategy was at the third level (i.e., moderate abstraction, see Figure 1), while the upper half of the distribution of strategies involved conceptually based problem solving requiring use of and reasoning about mathematical properties.

Similarly, problem-solving tools that appear in textbooks from the mid-1960s were used in consistently more abstract and conceptual ways. One example is the use of arrays in the teaching of multiplication. Arrays did not appear as a way to demonstrate multiplication until the late 1930s and were initially used in very static ways (e.g., Buswell, Brownell, & Lenore, 1938). Students of the 1930s were shown arrays of dots matching the two multiplicands and asked to count the marks or to create and count their own arrays based on the multiplicands to “prove” to themselves that the product was correct. In the 1990s textbooks, arrays were extended into an introduction to the concept of factoring and used as a way to demonstrate properties of multiplication (e.g., Ginsburg, 1999; Eicholz, 1991). For example, to facilitate understanding of factoring, students might be given a set of chips or other manipulatives and asked to investigate which rectangular arrays can be formed from the set of chips to determine the factors of a number. They could be asked to create arrays for the numbers 2 through 7 to determine all the multiplication facts for that product, 7.

Not only were students of the last two decades of the 20th century exposed to more abstract and conceptual strategies, they were also more often asked to solve problems using multiple strategies. This type of change suggests a curriculum that requires an increasingly advanced conceptual understanding of operations and numbers to shift between different problem representations and strategies. Figure 8 shows pages from two different 1999 fourth-grade textbooks summarizing the
various ways students can solve arithmetic problems. Pages such as these often encourage students to choose the strategy they prefer, but it is not uncommon for students to be asked to demonstrate more than one solution strategy for a problem or to use multiple strategies within a problem set.

III-C. Growth of arithmetic learning exercises using reasoning. A related way to look at changes in how students were asked to engage with arithmetic problems is the use of self-checking the logic behind the student’s answer, mental mathematics, and estimation. These approaches to problems require specific cognitive executive-functioning skills—novel problem-solving skills, inhibitory control, and use of working memory (e.g., Blair, 2006)—related to the mental retention of multiple pieces of information, representing problems in more than one way, and shifting attention between salient pieces of information—all key skills for basic reasoning.

Figure 9 shows the trend in the percentage of pages asking first- and fourth-grade students to self-check and examine the reasoning behind their work to check its logic and accuracy in solving a problem presented as a task. In the earliest periods, students were rarely asked to self-check, but this practice steadily increased until the 1930s and then fell precipitously over the middle part of the century until the mid-1970s, when its frequency began to steeply increase until it reached roughly 15% in the 1990s ($t = 2.43, p < .05$).

![Figure 9. Percent of pages with exercises requiring cognitive skills of self-check in textbooks, 1904–2000.](image-url)
As shown in Figure 10, the frequency of tasks requiring the use of mental mathematics follows a similar historical pattern. Solving problems without the use of paper and pencil grew from being presented on 17% of pages early in the century to a high of 33% by the mid-1920s. Following that, there is a decline in the percent use of this practice to nearly zero percent by the early 1970s, when its use starts to rebound, and by the end of the century, mental mathematics is found in tasks on one fifth of the pages. The strategies involved in mental mathematics also shifted from memory and recitation of fact earlier in the century to unrehearsed mental calculation by the end of the century.

As shown in Figure 11, tasks that asked students to first mentally estimate the answer to a problem by using what they knew about solving such problems in general in first- and fourth-grade textbooks grew steadily from relatively little use until the early 1960s to 17% of pages by the end of the century (2% to 17%, \( t = 3.02, p < .05 \)).

**III-D. Continuation of calculation and drill in basic arithmetic.** Finally, it should be noted that even though these reasoning skills expand significantly by the end of the century, skills in basic arithmetic (practice in using algorithms for calculation and drill) did decline—but did not vanish—in the most recent textbooks. Interestingly, while conceptual reasoning problem-solving strategies increased in textbooks, the percent of pages with algorithms on them also expanded throughout

![Figure 10](image-url). Percent of pages with exercises requiring cognitive skills of mental mathematics in textbooks, 1904–2000.
the century. However, the nature of algorithms changed over time in tune with the growing conceptual reasoning focus of materials noted previously. Later in the century, algorithms ceased to be rote, mechanical, step-by-step recipes for problem solving, and instead, textbook tasks asked the student to use more reasoned methods of solving problems. In spite of the qualitative change in algorithms, the number of problems requiring use of an algorithm has remained more or less constant over time, demonstrating that in absolute terms, late in the century students were given as much, or even more, opportunity to practice skills in basic arithmetic as their counterparts were 100 years ago. But, in the context of the changes described previously, since the mid-1960s textbooks have focused less on mechanical mathematics and rote drill, giving way to more abstract, conceptual, and reasoning-based approaches to mathematics and mathematical thinking.

DISCUSSION

For the first time since the beginning of systematic study of mathematics education in the United States, the results here represent a detailed empirical analysis of the mathematics curriculum reflected by a broad and representational sample of commonly used elementary school textbooks over the course of the 20th century throughout the nation. Furthermore, these findings are a result of a detailed and comprehensive coding of the material and reflect curricular change over time. The
content analysis of the textbook sample indicates that the distribution of topics and size of textbooks have changed substantially across generations of elementary school students in the United States. The textbooks used in the first two decades of the century were short and typically more balanced among basic arithmetic, geometry/measurement, and advanced arithmetic than in the midcentury. Against the backdrop of continual page growth in each period, the 1920s through midcentury marks a period in which the fundamental operations of basic arithmetic with whole numbers, taught primarily through continual application of the same mechanical computational approach, came to dominate the expanding content of textbooks. This was particularly the case for first through third grades, and is also evident in fourth-through sixth-grade books.

From the mid-1960s, however, several distinctly new content trends emerge over the next 4 decades. First, the number of pages dedicated to basic arithmetic becomes stable as its proportion of the content in the text drops. Increasingly, the remainder of the textbooks were made up of substantially more geometry for first-through third-grade textbooks and more advanced arithmetic for fourth-through sixth-grade textbooks, such as more complex uses of the four basic operations and representations of numbers including decimals, fractions, and ratios, as well as percents and proportions. This content often builds on the basic concepts, ideas, and skills learned related to operations on whole numbers and is traditionally taught after basic arithmetic is covered. Second, more content on geometry and measurement was added including geometric concepts; measurement and units of measure; time; and perimeter, area, and volume. Third, starting in the mid-1960s the introduction of the concepts, ideas, and skills from more advanced content occur in textbooks for students in earlier grades. And last, a substantial and increasing amount of material on reasoning consisting of pattern solving, informal algebra, informal geometry, probability, and grouping/categorization is found in textbooks for all primary grades.

The cognitive assessment shows that starting in the mid-1960s, the kinds of problem-solving strategies, the abstract conceptualization of problem-solving strategies for basic arithmetic, and the use of effortful reasoning approaches increased. At the same time, textbooks of the last 35 years decreasingly required rote drill and mechanical application of algorithms and memorization of arithmetic facts. Although there are observable subtrends from the mid-1960s onward, such as the short slump during the back-to-basics curricular reforms of the mid-1970s, and subtle differences between the early and later grades of elementary school, for the most part these content changes and increased cognitive demands intensify for all elementary schooling throughout the rest of the century.

Does the sum total of these changes yield a more mathematically challenging and cognitively demanding textbook than that used before the mid-1960s? The content growth from the mid-1960s onward was both mathematically and cognitively different from the expansion earlier in the century. Through the 1950s, first-through fourth-grade students used texts that rarely, if ever, presented more complex uses of the four basic operations and more complex representations of numbers.
These textbooks also did not include content such as statistics and probability that was found in textbooks later in the century. In contrast, end-of-century textbooks introduce these ideas and related tasks in first grade and even in kindergarten. Given the growth of these advanced topics and the substantial growth in reasoning at successively earlier grades, a case can be made that U.S. elementary school mathematics curricula of the late 20th century is both mathematically and cognitively more demanding.

The trends towards greater use of reasoning about mathematics and more conceptually oriented problem solving presented with very young students will likely stir again the debate over whether or not current curricula—as reflected in end-of-century textbooks—is better for learning mathematics than a focus mostly on computational skills. Although this debate is beyond the scope of this study, the results raise interesting new directions for research into mathematics education that can contribute to the debate. What is certain is that over the last several decades, students are increasingly asked to work with content that engages them in activities such as estimation and developing multiple solution strategies for problems that demand effortful cognitive skills closely associated with executive functioning (Bull & Scerif, 2001; Espy et al., 2004).

The evidence from the assessment of problem solving in basic arithmetic sections of textbooks is particularly compelling on this point. Later in the century, the young learner is required to not only master more problem-solving strategies but also to understand the mathematical properties underlying problem-solving strategies in an abstract fashion that is transferrable to new problems. Starting with textbooks for students as young as 5 or 6, decreased emphasis on mechanical application of set problem-solving procedures and increased focus on understanding and applying the properties of mathematics in multiple ways is a qualitative change—one that takes the cognitive demands of the curriculum in an entirely different direction than what was presented in textbooks to learners prior to the mid-1960s. By the 1990s, rather than asking students to mechanically apply problem-solving “recipes,” textbooks challenge young learners to use their modest knowledge of mathematical properties to engage in conceptual mathematical problem solving.

From a cognitive psychological perspective, the increased focus on strategies reflects an increase in mathematics activities that exercise students’ general reasoning abilities. More recent textbooks not only expose students to an increasing number of more abstract problem-solving strategies, but also show an overall increase in mathematics activity that requires students to estimate, use multiple strategies, or perform self-checks of their work. After the mid-1960s these changes occur in textbooks with a concurrent decline in the average numbers of mechanical algorithms and problems per page, suggesting the curriculum refocused from mechanical and rote activity to more abstract and flexible reasoning-based mathematical thinking. In essence, the mathematics curriculum has come to emphasize effortful cognitive processing, or reasoning, over memorization and drill, creating greater cognitive demands, particularly on executive-functioning capabilities. Although these concepts could have continued to have been taught in more basic
and mechanical ways, textbooks increasingly provided opportunities for teachers to utilize new problem-solving strategies, which made it more likely that as the changes became deeply embedded in the textbooks, they would be adopted in growing numbers of classrooms.

This shift is particularly noteworthy in contrast to other plausible historical scenarios. The elementary mathematics curriculum, as reflected in textbook content, could have continued on its slow course toward full dominance by basic arithmetic, with ever more focus on the application of problem-solving recipes and a utilitarian approach to using mathematics. Or it could have easily remained the same since the period from 1956–1963. Instead, from the mid-1960s, the historical path it took opened up a number of avenues of speculation on the historical influence behind these results (Kliebard, 1995; Stanic & Kilpatrick, 2003).

Although a full analysis of the historical forces behind these curricular results is beyond the scope of this study, it is useful to suggest some likely causes of the trends that can be examined in future historical research. The overall outlines of the historical trends documented here and their possible causes will not come as a surprise to the mathematics education community in the United States, as many of its members have been involved in reform efforts from the mid-1960s onward. A likely important factor preceding the textbook changes that began in the mid-1960s was the rise of New Math. The initial New Math curricular reform movement was intensified through the formation of the School Mathematics Study Group (SMSG) in 1958, which was itself precipitated by the educational crisis created over the Soviet Sputnik launch (Fey & Graeber, 2003). The reactionary “back-to-basics” movement of the mid-1970s retarded New Math’s development, although innovation resumed with the curricular and standards reform activities of the National Council of Teachers of Mathematics (NCTM) starting in the 1980s. The Agenda for Action in 1980 and then the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) attempted to reform mathematics education with an emphasis on conceptual understanding and problem solving— informsed by a constructivist understanding of how children learn—with less rote learning of mathematics facts (McLeod, 2003). The results here suggest that these efforts have made for changes in U.S. elementary school mathematics textbooks of the last several decades. And behind these results have been many debates and controversies about how mathematics education should proceed, which will, no doubt, continue into the future.

Other factors related to mathematics education were likely also to have played a role. Two prominent ones are the rise of professional research on American school mathematics education in the United States and elsewhere (a result of which was the founding of this journal), as well as the rise of the cognitive revolution in psychology (Lester & Lambdin, 2003; Miller, 2003). So too, changes in textbook publishing such as the collapse and consolidation of the textbook industry probably has had an effect. And lastly, a growing sociological literature on historical change in the content of other academic subjects suggests causal factors on a more global—as opposed to national or local—level (e.g., Drori, Meyer, Ramirez, & Schofer, 2003; Young & Muller, 2007).
The trends before the mid-1960s are also interesting grist for speculation. From the perspective of the content and challenge of the 1990s textbook, the period from 1956–1963 represents a low point in the difficulty of the elementary school mathematics textbook to which the later reform efforts reacted. How and why did relevant intellectual, political, and administrative factors produce a curriculum based heavily on basic arithmetic taught through rote exercises? Two important factors to consider were how American schools reacted to rising enrollments in this period and to the pedagogical outcomes of the debate between vocationalism and academic development from the 1920s onward through midcentury (Angus & Mirel, 2003). Additionally, during this period, reactions to a perceived lack of mathematical skills among U.S. soldiers during World War II put in place motivations for the reform period that burst forth in the mid-1960s (Garrett & David, 2003).

The historical perspective taken here shows the degree to which mathematics textbooks can, and have, changed. To date, influential conclusions about the American mathematics curriculum have been based on cross-sectional analyses of textbooks and classroom implementation of curriculum, usually from cross-national studies (e.g., Schmidt, McKnight, Valverde, Houang, & Wiley, 1997). The usual speculation from these studies is that, historically, the American mathematics curriculum has become broader, and this broadness is marked by, and may even cause, a lack of depth and complexity of content. The results here support a contrasting image of a broad as well as deep curriculum as reflected in textbooks since the mid-1960s. Contrary to what is often assumed, the results about the relative consistency of textbook material within time periods suggests that the way in which textbooks are developed and marketed in the United States has not necessarily produced a chaotic written curriculum in recent decades, contrary to what others have argued (e.g., Tyson-Bernstein, 1988). Moreover, the results here will assist mathematics education researchers to better inform national debates about the state of mathematics education.

These results also raise important questions about the relation of schooling to children’s cognitive development. The changes in the content of the curriculum after the mid-1960s were mirrored in changing conceptualizations of the cognitive capabilities of young children as reflected in research on child psychological development. In the 1960s, behaviorism with its emphasis on stimulus-response associations as the basis for learning was supplanted by an interest in cognitive processes and Piagetian notions of how children acquire and use knowledge. As a result, a major trend in research on infant and child cognition throughout the last quarter of the 20th century has been the demonstration of specific cognitive capabilities at increasingly younger ages. In contrast to the once-dominant Piagetian paradigm, levels of cognitive ability previously thought to be unattainable until middle childhood have been shown (Blair & Razza, 2007; Bull & Scerif, 2001; Espy et al., 2004) to be present in infancy and early childhood. Similarly there is growing evidence (Eslinger, Flaherty-Craig, & Benton, 2004) that environmentally stimulated cognitive development is sustained through early adolescence, and
perhaps significantly longer in the life span. So too, it is likely that the changes in the content of the mathematics curriculum and in research on child psychological development (e.g., von Glasersfeld, 1991) are related and mutually reinforcing. That is, given a growing recognition of the diverse cognitive capabilities of young children, mathematics education began to incorporate activities designed specifically to exercise and promote these abilities, including basic reasoning abilities. In this way, intellectual and historical forces began to identify and emphasize through educational content certain aspects of cognitive functioning over others, perhaps playing an important role in shaping child cognitive development. The implications of this changing emphasis (i.e., reasoning over memorization) remain to be determined, but it would appear to be related to a broad historical trend in mental ability in the general population. As shown in the analysis of mental test performance over the 20th century, ability levels of aspects of cognitive functioning that support general reasoning and conceptual problem-solving processes have been increasing monotonically, and the results here suggest that schooling plays a central role in this change (Blair, Gamson, Thorne, & Baker, 2005; Flynn, 2007; Martinez, 2000). One caveat is that these results are about textbooks as an indication of the formal curriculum, and they do not necessarily reflect the enacted curriculum in classrooms. Although it is reasonable to assume some connection between textbook content and the enacted curriculum, there is no empirical way to discern the level of fidelity to mathematics textbooks that occurred in classrooms of the past, or even of the recent past. And as briefly reviewed previously, there is research suggesting that textbook fidelity across teachers can vary considerably depending on a host of factors (Remillard, Herbel-Eisenmann, & Lloyd, 2009; Stein, Remillard, & Smith, 2007). What the results here do represent though, is the written record of major curricular materials available to teachers and students in classrooms across the nation over the past 100 years. Lacking a source of centralized national control of curriculum with accompanying documentation, these textbooks are the only available historical record of U.S. elementary school formal curricula in mathematics.

REFERENCES


Elementary School Textbook Analysis


**Authors**

**David Baker**, The Pennsylvania State University, Department of Education Policy Studies, 300 Rack-ley Building, University Park, PA 16802; dpb4@psu.edu

**Hilary Knipe**, New York University, Department of Applied Psychology, 239 Greene St. East Bldg 500, New York, NY 10003; hknipe@gmail.com

**John Collins**, The Pennsylvania State University, Department of Education Policy Studies, 300 Rack-ley Building, University Park, PA 16802; jmc677@psu.edu
Juan Leon, The Pennsylvania State University, Department of Education Policy Studies, 300 Rackley Building, University Park, PA 16802; jjl292@psu.edu

Eric Cummings, Cumberland University, School of Education, 104 Bone Hall, Lebanon, TN 37087; ecummings@cumberland.edu

Clancy Blair, New York University, Department of Applied Psychology, 239 Greene St. East Bldg 500, New York, NY 10003; cbb5@nyu.edu

David Gamson, The Pennsylvania State University, Department of Education Policy Studies, 300 Rackley Building, University Park, PA 16802; dag17@psu.edu

Accepted Month day, year
# APPENDIX A

Selected Mathematics Textbooks and Textbook Series by Historical Period

<table>
<thead>
<tr>
<th>Period</th>
<th>Title</th>
<th>Publisher</th>
<th>Years published</th>
</tr>
</thead>
<tbody>
<tr>
<td>1904–1921 (23 books)</td>
<td>Southworth Stone Arithmetic</td>
<td>BH Sanborn</td>
<td>1904–1908</td>
</tr>
<tr>
<td></td>
<td>Practical Arithmetic</td>
<td>Ginn and Co.</td>
<td>1905</td>
</tr>
<tr>
<td></td>
<td>Wentworth-Smith Mathematical Series:</td>
<td>Ginn and Co.</td>
<td>1907–1914</td>
</tr>
<tr>
<td></td>
<td>New Elementary Arithmetic</td>
<td>Ginn and Co.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>First Journeys in Numberland</td>
<td>Scott Foresman</td>
<td>1911</td>
</tr>
<tr>
<td></td>
<td>The Stone-Millis Arithmetic</td>
<td>BH Sanborn</td>
<td>1910–1921</td>
</tr>
<tr>
<td></td>
<td>School Arithmetics</td>
<td>Ginn and Co.</td>
<td>1920</td>
</tr>
<tr>
<td></td>
<td>The New Stone-Millis Arithmetics</td>
<td>BH Sanborn</td>
<td>1910–1921</td>
</tr>
<tr>
<td>1924–1931 (15 books)</td>
<td>A Child’s Book of Number</td>
<td>B.H. Sanborn</td>
<td>1924</td>
</tr>
<tr>
<td></td>
<td>The Smith-Burdge Arithmetic Primary</td>
<td>Ginn and Co.</td>
<td>1926</td>
</tr>
<tr>
<td></td>
<td>Strayer-Upton Arithmetic</td>
<td>American Book Co.</td>
<td>1924–1931</td>
</tr>
<tr>
<td></td>
<td>Problem and Practice Arithmetic</td>
<td>Ginn and Co.</td>
<td>1929–1931</td>
</tr>
<tr>
<td></td>
<td>Walks and Talks in Numberland</td>
<td>Ginn and Co.</td>
<td>1929</td>
</tr>
<tr>
<td></td>
<td>The Problem and Practice Arithmetics</td>
<td>Ginn and Co.</td>
<td>1929–1931</td>
</tr>
<tr>
<td></td>
<td>Number Games and Stories</td>
<td>Houghton Mifflin</td>
<td>1930</td>
</tr>
<tr>
<td>1932–1948 (22 books)</td>
<td>Number Stories</td>
<td>Scott Foresman</td>
<td>1932–1947</td>
</tr>
<tr>
<td></td>
<td>Daily-Life Arithmetics</td>
<td>Ginn and Co.</td>
<td>1938</td>
</tr>
<tr>
<td></td>
<td>Living Arithmetic</td>
<td>Ginn and Co.</td>
<td>1943–1951</td>
</tr>
<tr>
<td></td>
<td>Study Arithmetics</td>
<td>Scott Foresman</td>
<td>1934–1948</td>
</tr>
<tr>
<td></td>
<td>Jolly Numbers</td>
<td>Ginn and Co.</td>
<td>1937–1959</td>
</tr>
<tr>
<td></td>
<td>Growth in Arithmetic</td>
<td>World</td>
<td>1952–1956</td>
</tr>
<tr>
<td></td>
<td>Our Number Workshop</td>
<td>Scott Foresman</td>
<td>1952–1961</td>
</tr>
<tr>
<td></td>
<td>Growth in Arithmetic</td>
<td>Harcourt Brace and World</td>
<td>1952–1962</td>
</tr>
<tr>
<td></td>
<td>Arithmetic We Need</td>
<td>Ginn and Co.</td>
<td>1955–1963</td>
</tr>
<tr>
<td></td>
<td>Jolly Numbers</td>
<td>Ginn and Co.</td>
<td>1937–1959</td>
</tr>
<tr>
<td></td>
<td>Growth in Arithmetic</td>
<td>Harcourt Brace and World</td>
<td>1952–1962</td>
</tr>
<tr>
<td></td>
<td>Numbers We Need</td>
<td>Ginn and Co.</td>
<td>1955–1963</td>
</tr>
<tr>
<td></td>
<td>Making Sure of Arithmetic</td>
<td>Silver Burdett</td>
<td>1946–1958</td>
</tr>
<tr>
<td></td>
<td>Jolly Numbers</td>
<td>Ginn and Co.</td>
<td>1937–1959</td>
</tr>
<tr>
<td></td>
<td>Arithmetic We Need</td>
<td>Ginn and Co.</td>
<td>1955–1963</td>
</tr>
<tr>
<td></td>
<td>Seeing Through Arithmetic (A)</td>
<td>Scott Foresman</td>
<td>1956–1963</td>
</tr>
<tr>
<td></td>
<td>Arithmetic We Need</td>
<td>Ginn and Co.</td>
<td>1955–1963</td>
</tr>
<tr>
<td></td>
<td>Moving Ahead in Arithmetic</td>
<td>Holt, Rinehart, and Winston</td>
<td>1963</td>
</tr>
<tr>
<td></td>
<td>Seeing Through Arithmetic</td>
<td>Scott Foresman</td>
<td>1966–1968</td>
</tr>
<tr>
<td></td>
<td>Elementary School Mathematics</td>
<td>Addison-Wesley</td>
<td>1968–1971</td>
</tr>
<tr>
<td></td>
<td>Addison-Wesley Mathematics</td>
<td>Addison-Wesley</td>
<td>1985–1995</td>
</tr>
<tr>
<td></td>
<td>Silver Burdett Ginn Mathematics</td>
<td>Silver Burdett</td>
<td>1999–2001</td>
</tr>
</tbody>
</table>
## APPENDIX B

List of Courses of Study and Other Documents Examined for Textbook Information

<table>
<thead>
<tr>
<th>City/State/Organization</th>
<th>Title/Focus/Subject area</th>
<th>Date(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brookline (MA)</td>
<td>Course of Study</td>
<td>1895</td>
</tr>
<tr>
<td>Philadelphia</td>
<td>Course of Study in Arithmetic</td>
<td>1898</td>
</tr>
<tr>
<td>East St. Louis Public Schools</td>
<td>Detailed Course of Studies for Grade Courses in Arithmetic</td>
<td>1914</td>
</tr>
<tr>
<td>Commonwealth of MA</td>
<td>A course of Study in Arithmetic</td>
<td>1916</td>
</tr>
<tr>
<td>Virginia Public State Schools</td>
<td>Survey of the State School System</td>
<td>1920</td>
</tr>
<tr>
<td>Berkeley, CA</td>
<td>Arithmetic: Course of Study</td>
<td>1923</td>
</tr>
<tr>
<td>Baltimore County</td>
<td>Course of Study in Arithmetic</td>
<td>1926</td>
</tr>
<tr>
<td>Elementary Schools of Kansas</td>
<td>New Course of Study (arithmetic and history)</td>
<td>1927</td>
</tr>
<tr>
<td>City of Baltimore</td>
<td>Arithmetic: Course of Study for Grades Four, Five, and Six</td>
<td>1929</td>
</tr>
<tr>
<td>NSSE Yearbook</td>
<td>“The Arithmetic Curriculum”</td>
<td>1930</td>
</tr>
<tr>
<td>Commonwealth of MA</td>
<td>Course of Study for Arithmetic in Elementary Schools</td>
<td>1931</td>
</tr>
<tr>
<td>Kansas</td>
<td>Course of Study in Arithmetic for the Elementary Schools</td>
<td>1937</td>
</tr>
<tr>
<td>Louisiana State department of education</td>
<td>Course of study in arithmetic</td>
<td>1943</td>
</tr>
<tr>
<td>NCTM</td>
<td>“Curriculum Standards for Grades K–4”</td>
<td>1989</td>
</tr>
<tr>
<td>NCTM</td>
<td>Curriculum and Evaluation Standards for School Mathematics</td>
<td>1989</td>
</tr>
<tr>
<td>NCTM</td>
<td>Professional Standards for Teaching Mathematics</td>
<td>1991</td>
</tr>
<tr>
<td>Mathematics Teacher</td>
<td>“How Reform Secondary Mathematics Textbooks Stack up against NCTM’s Principles and Standards”</td>
<td>2001</td>
</tr>
</tbody>
</table>